



# Computational Geometry

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# Terminology

- Point
- Line
- Line segment
- Ray

# Line – Line Intersection

- Given two lines, calculate their intersection.

$$y = ax + c \qquad y = bx + d$$

$$P \left( \frac{d - c}{a - b}; a \frac{d - c}{a - b} + c \right)$$

- Can do a similar thing for line segments; must just check whether intersection lies within the range of segment.

# CCW Function

- Determine whether a given set of three ordered points go counter-clockwise or clockwise.

Use this:

$$(x_2 - x_1)(y_3 - y_1) - (y_2 - y_1)(x_3 - x_1)$$

If 0, then collinear

If positive, then “left turn”

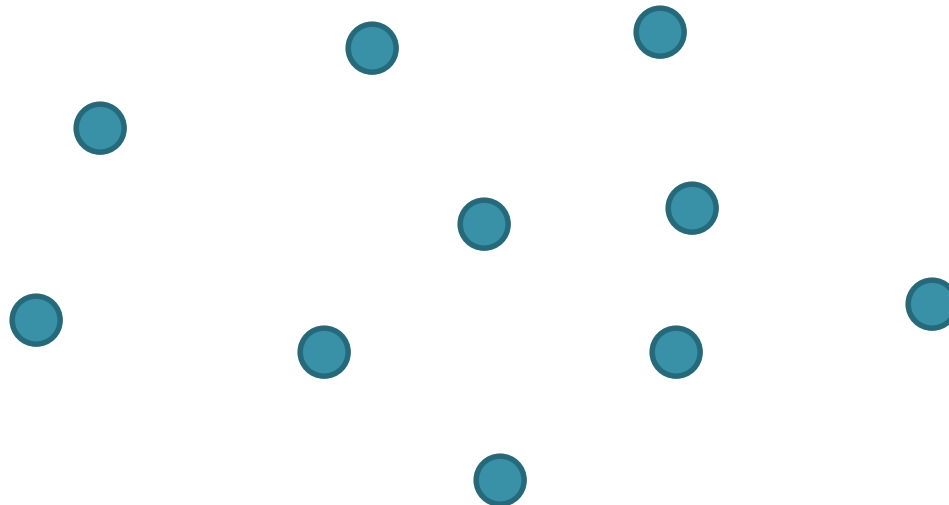
If negative, then “right turn”

# Convex Hull

- Jarvis March (“Gift Wrapping” algorithm)
  - Runs in  $O(nh)$  (Worst case:  $O(n^2)$  )
- Graham Scan
  - Runs in  $O(n \log n)$

# Jarvis March

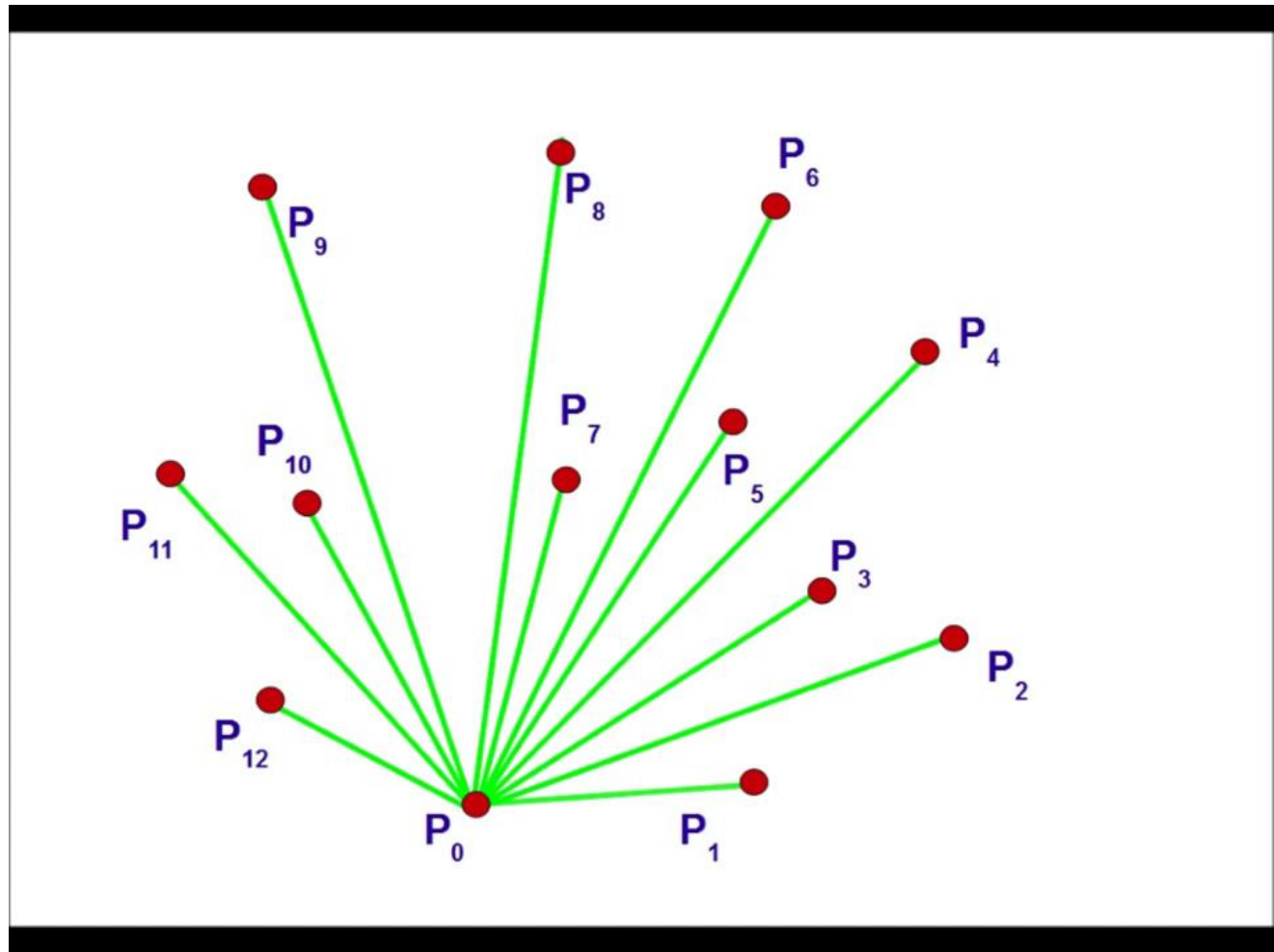
- Pick a point on convex hull.
- Loop through all points and find the one that forms the minimum sized anticlockwise angle off the horizontal axis from the previous point.
- Continue until you encounter the first point.



# Graham Scan

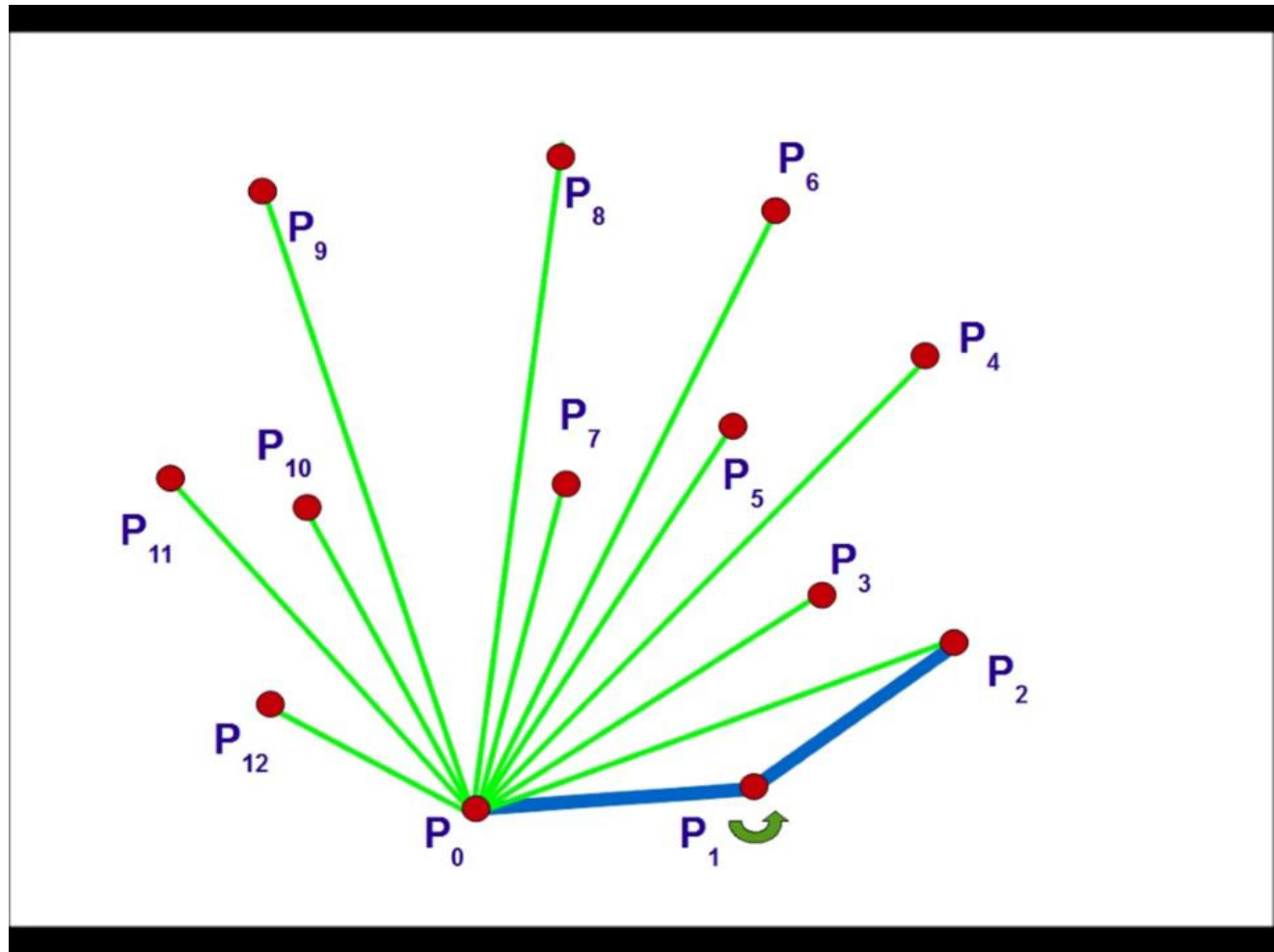
- Pick a point on convex hull.
- Sort all other points angularly around this point.
- Add the first two points to the hull.
- For every next point, check if that point, along with the preceding two, form a “right turn” or a “left turn”.
- If “right turn”, remove second last point, if “left turn”, move on to next point.
- Continue until you encounter the first point.

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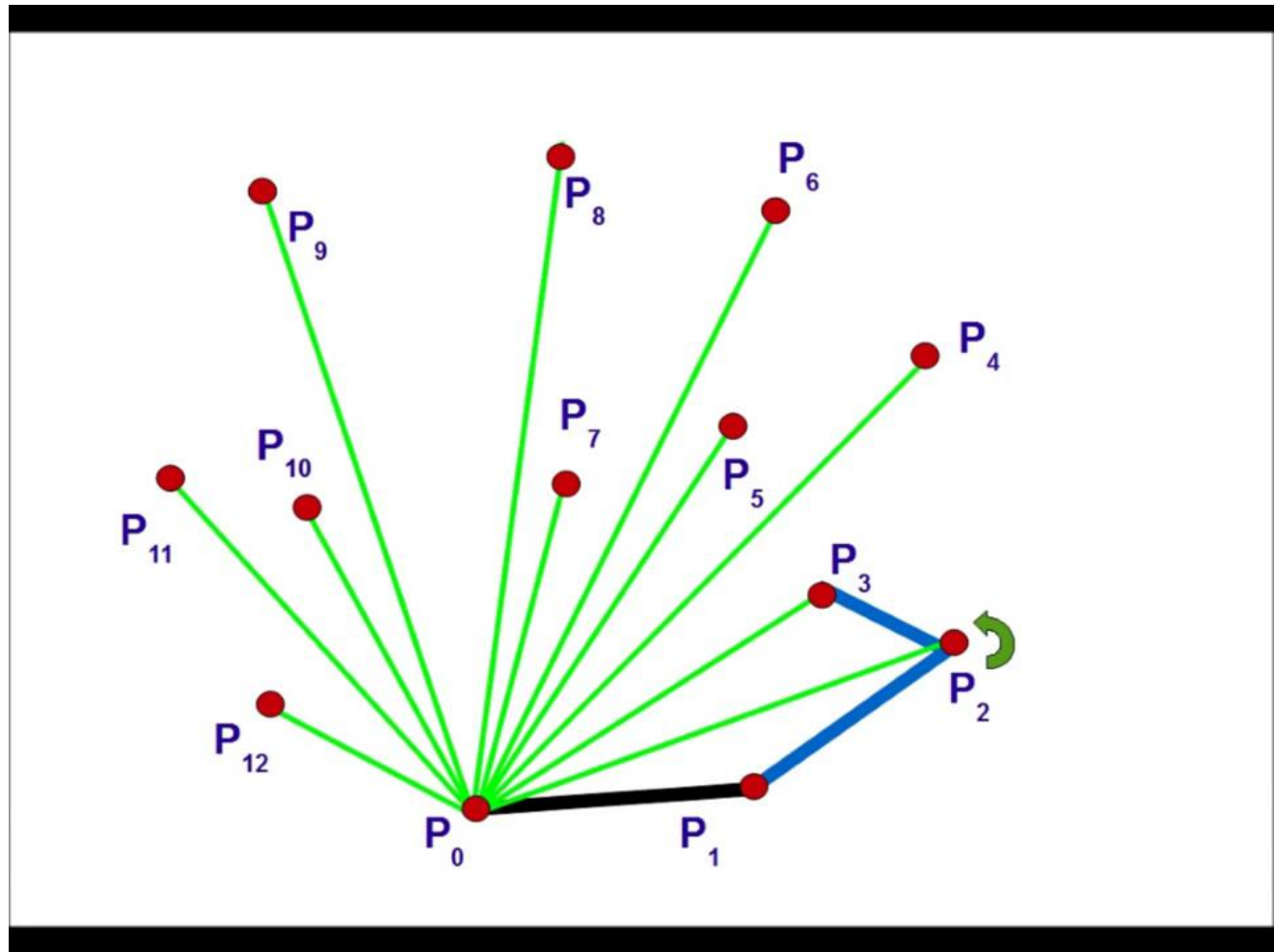




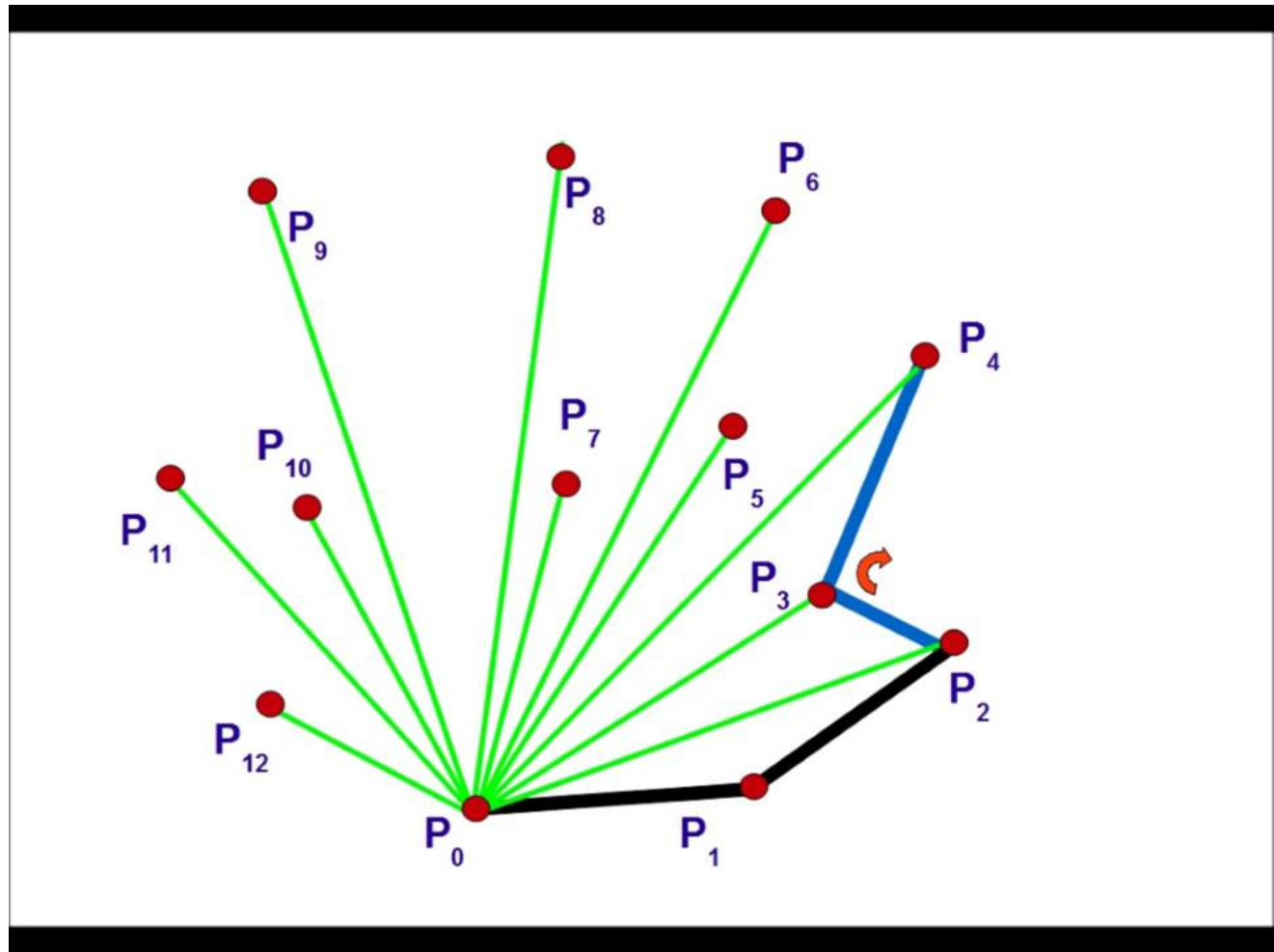
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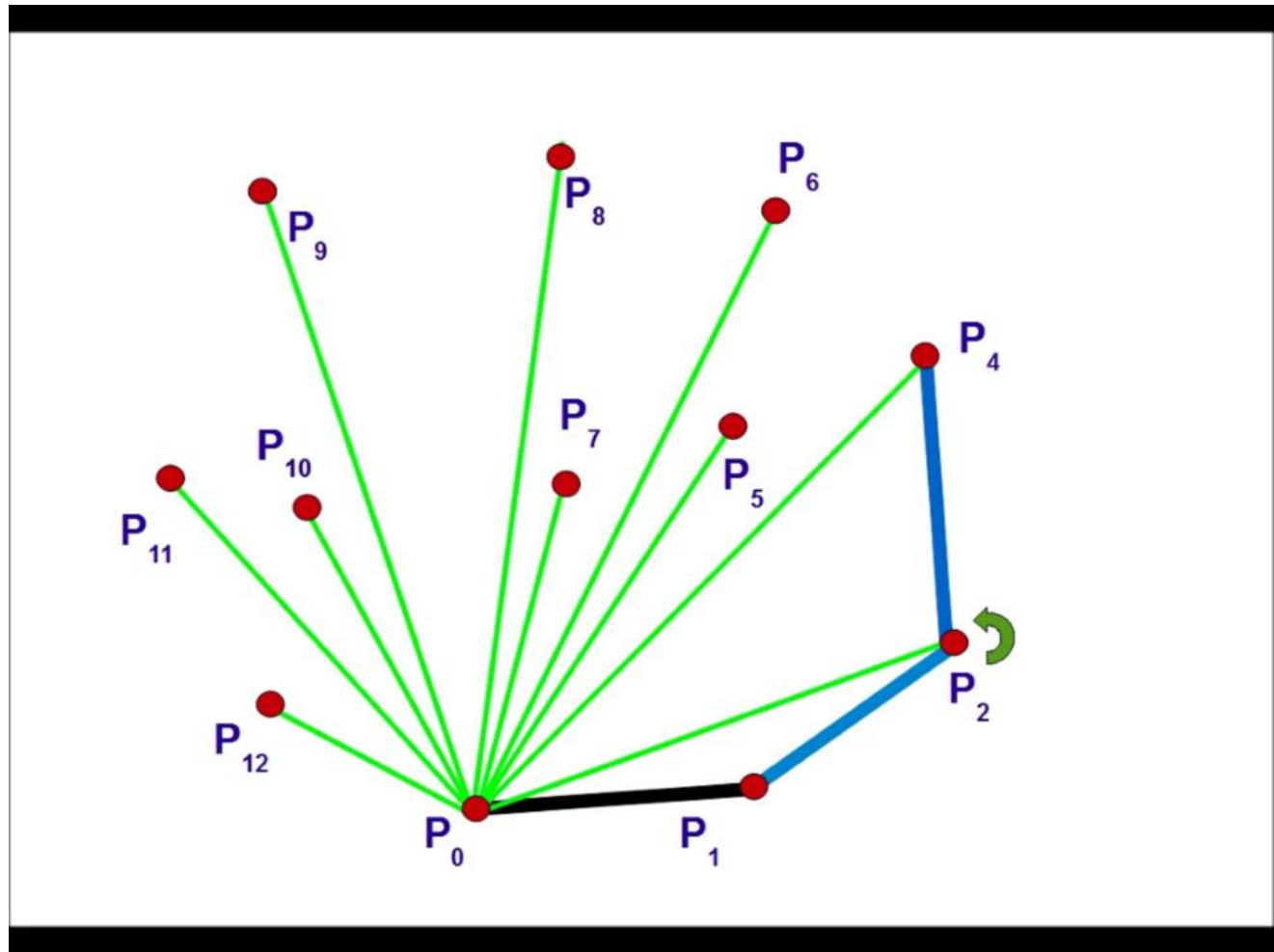
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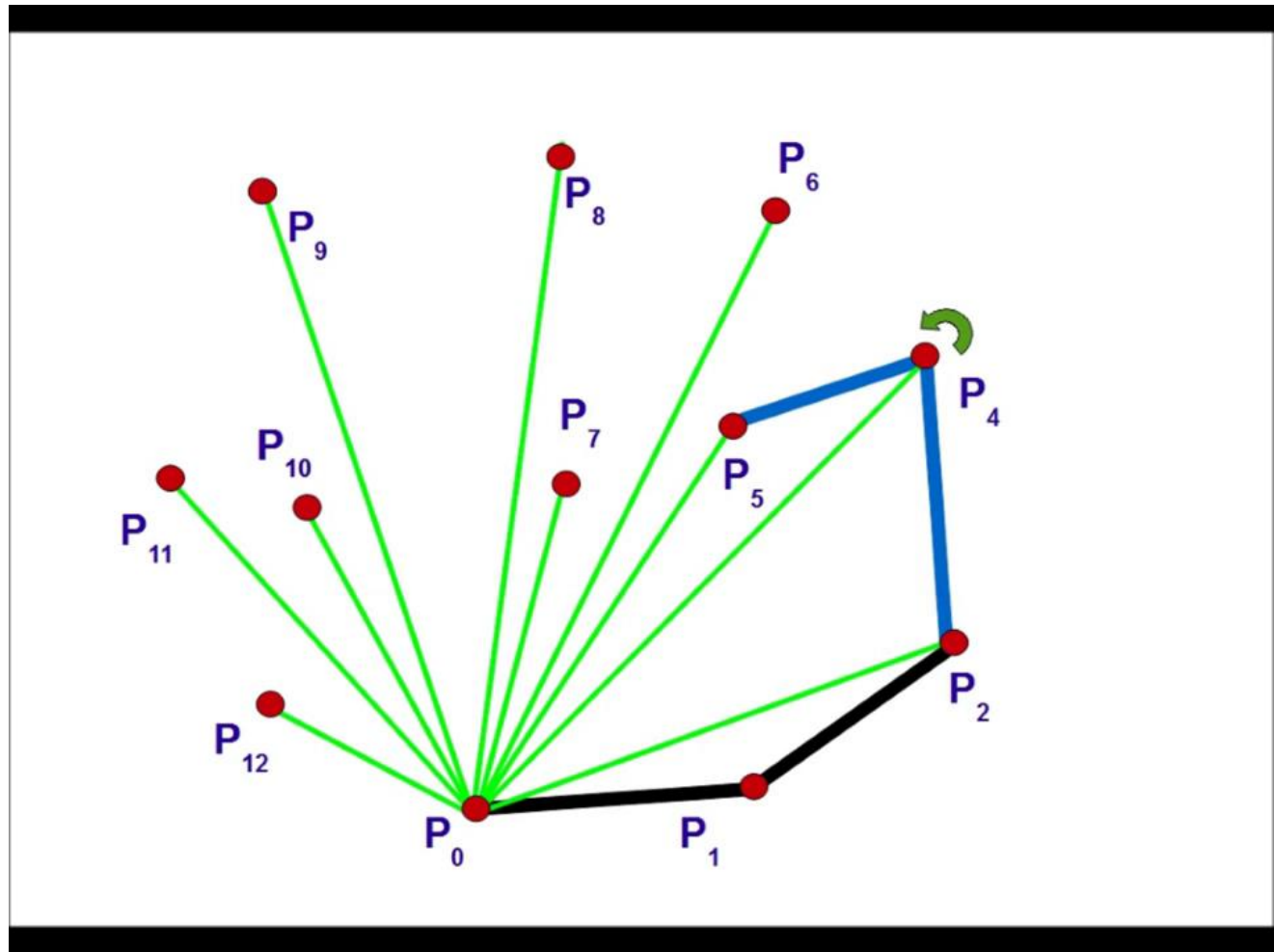
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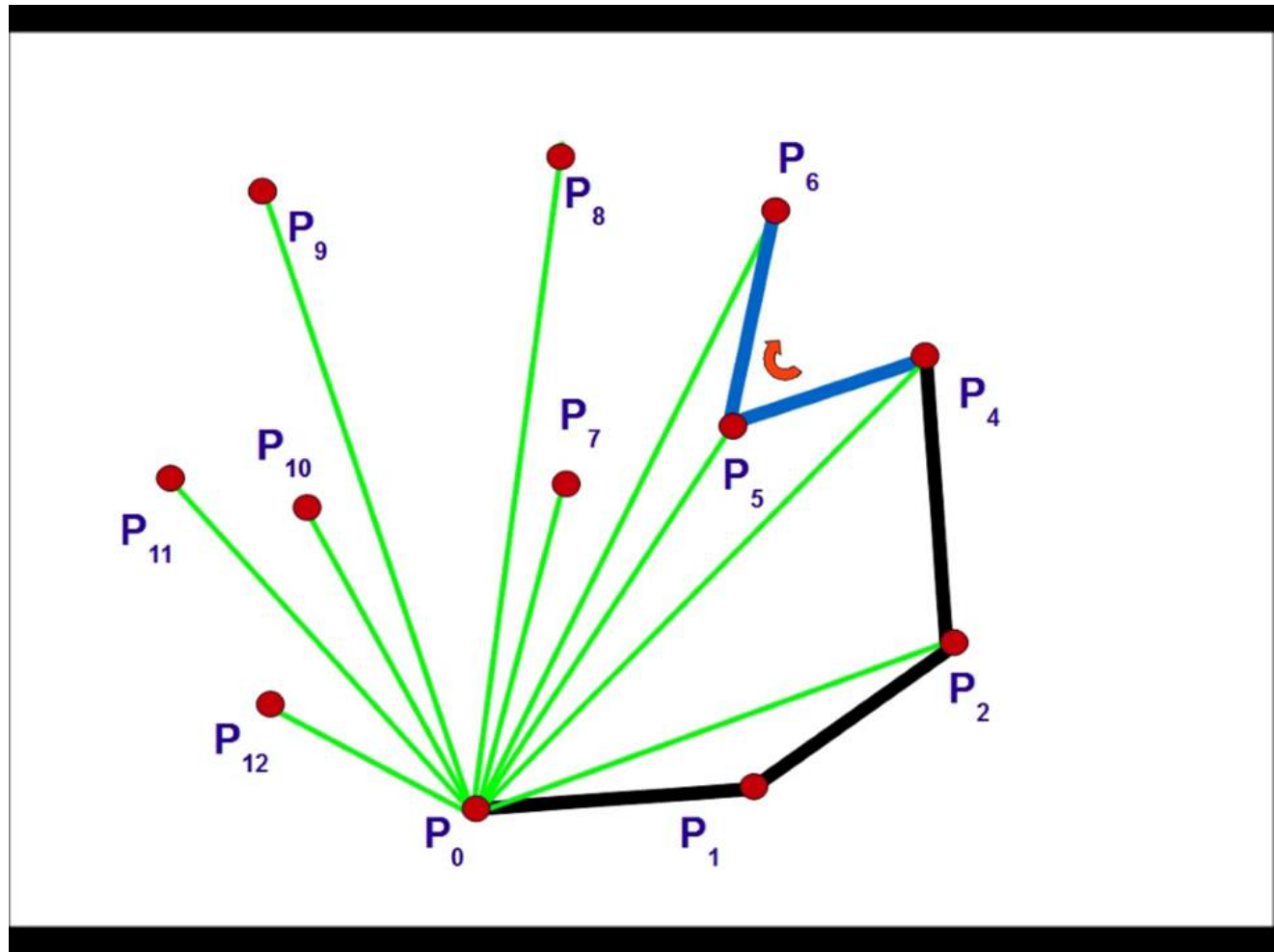
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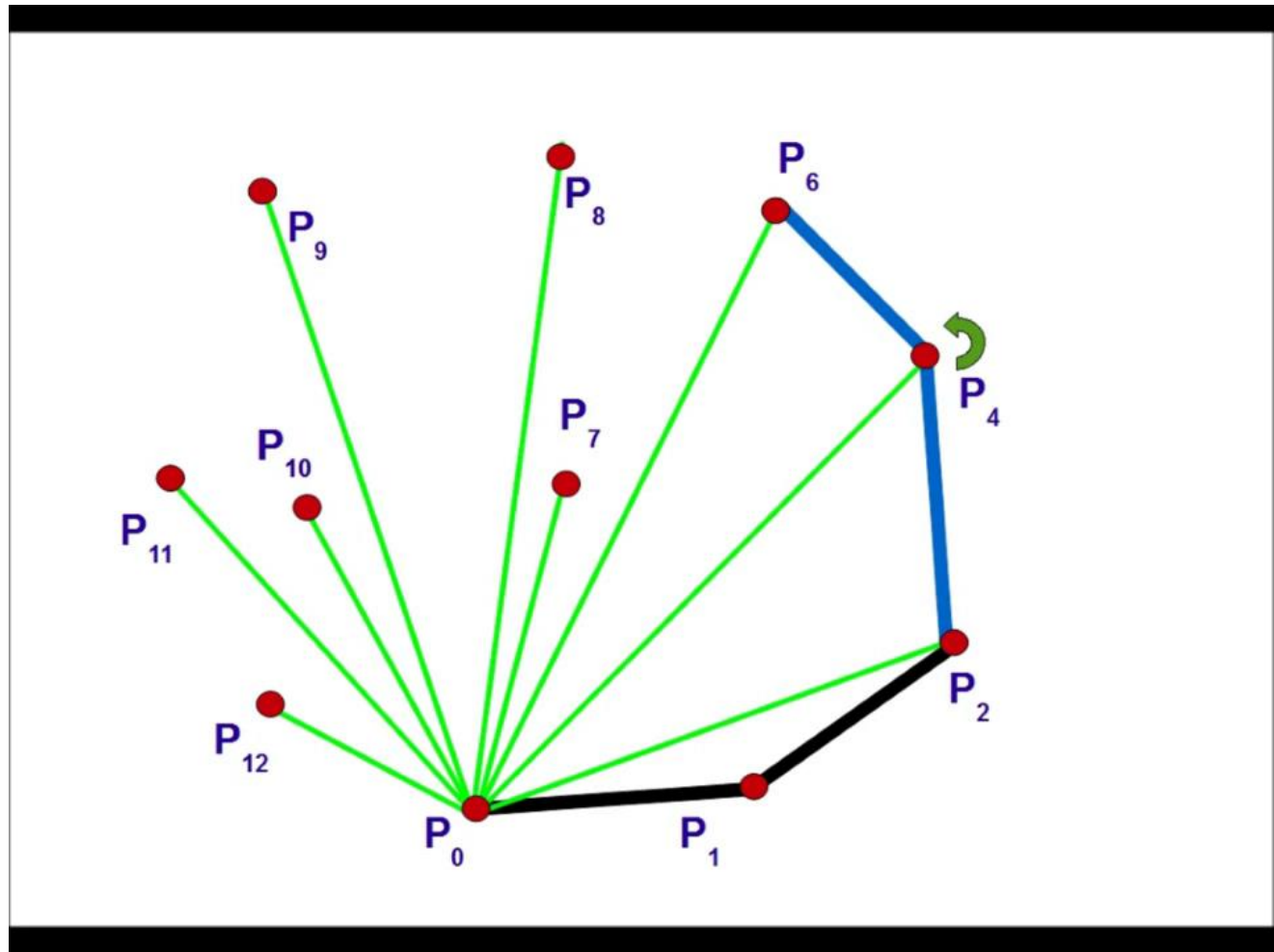
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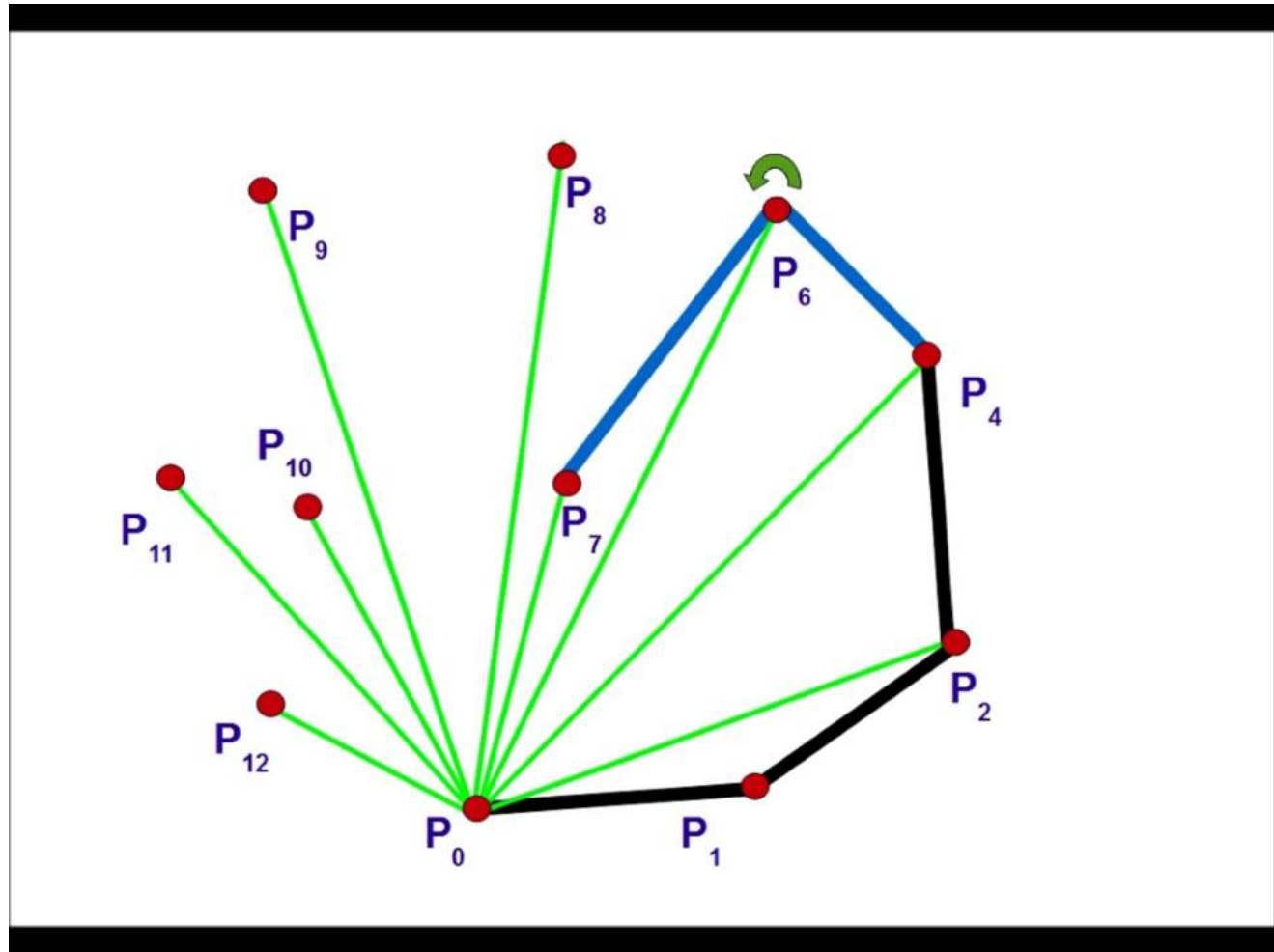
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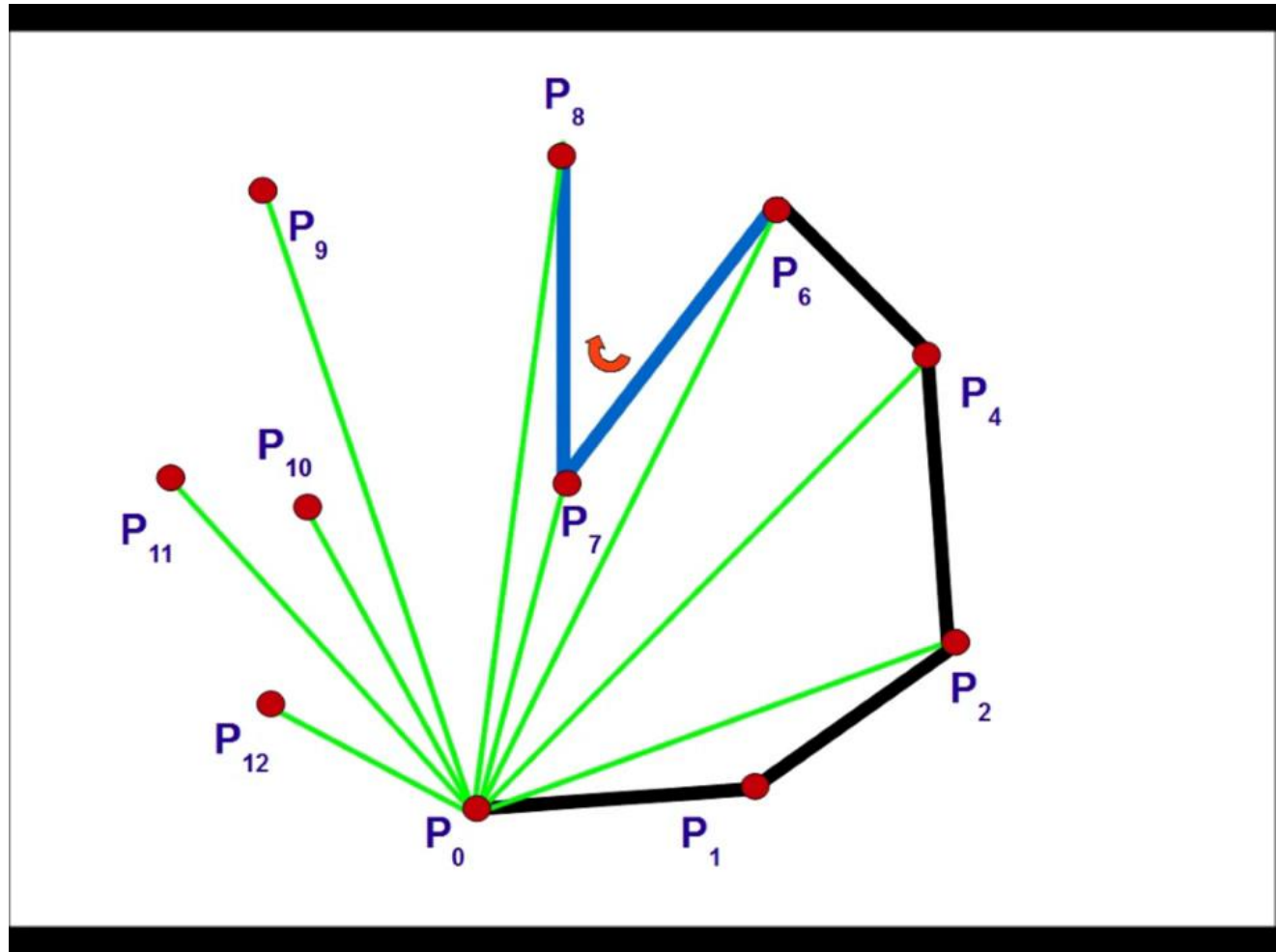


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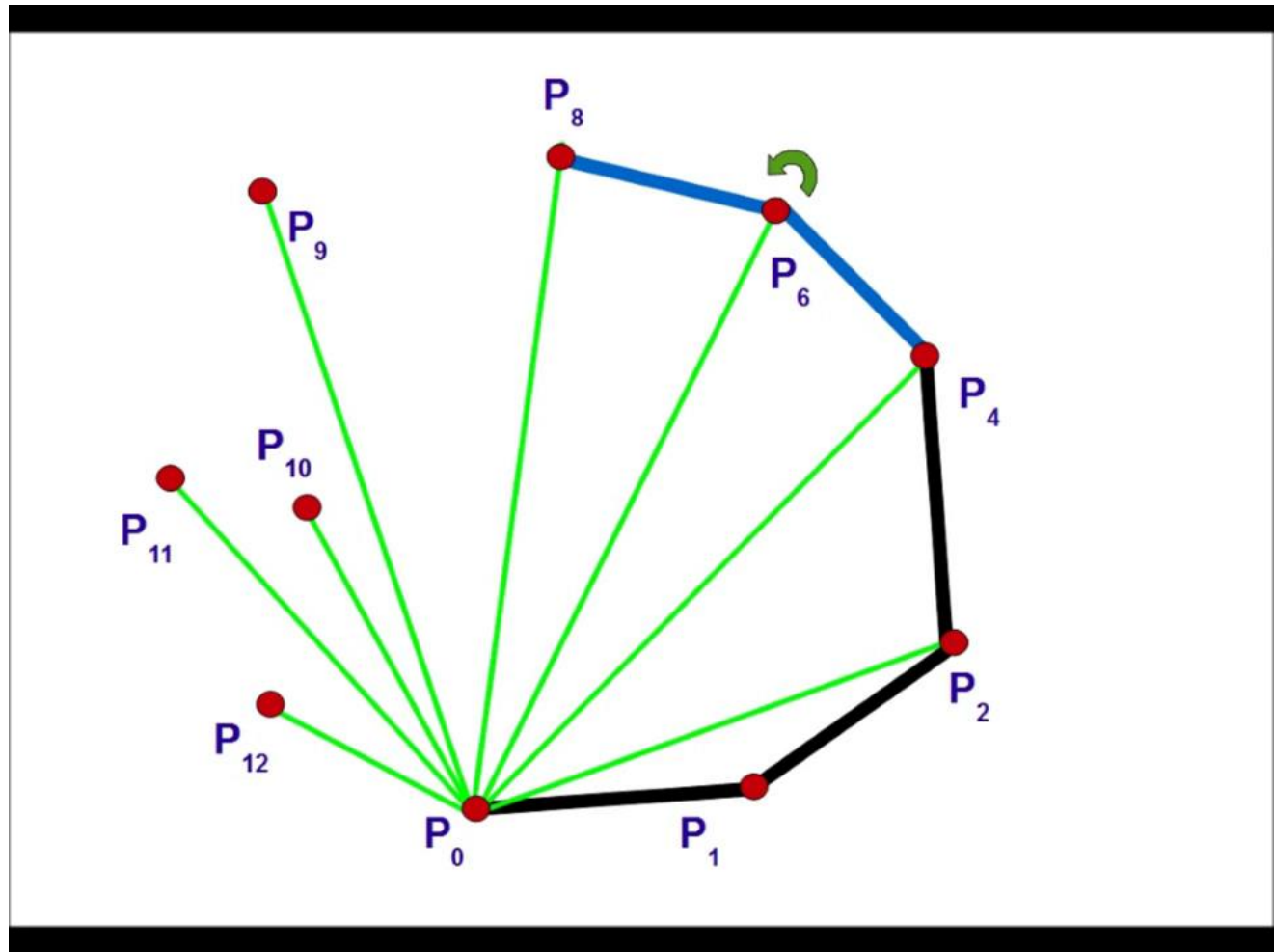




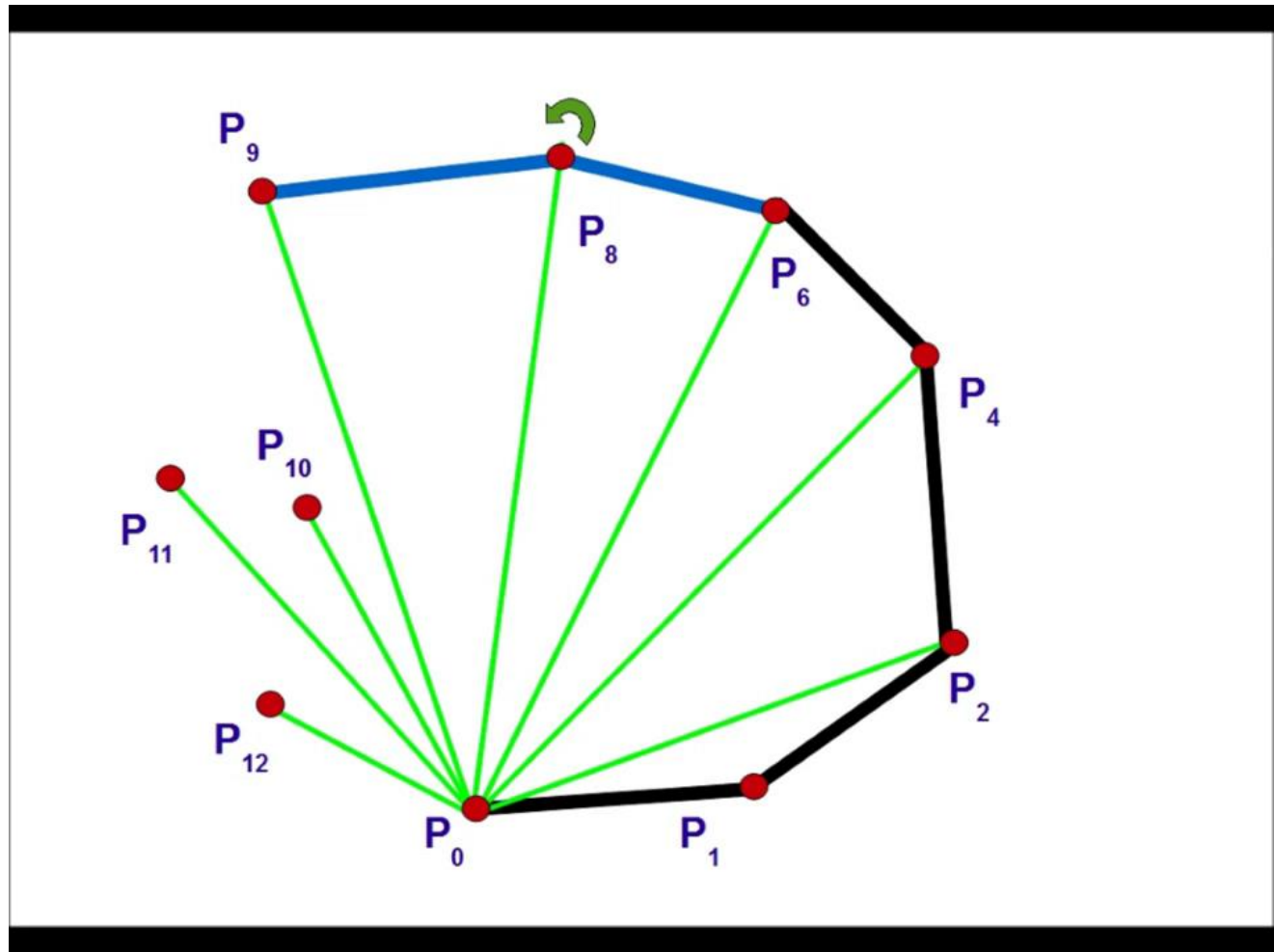
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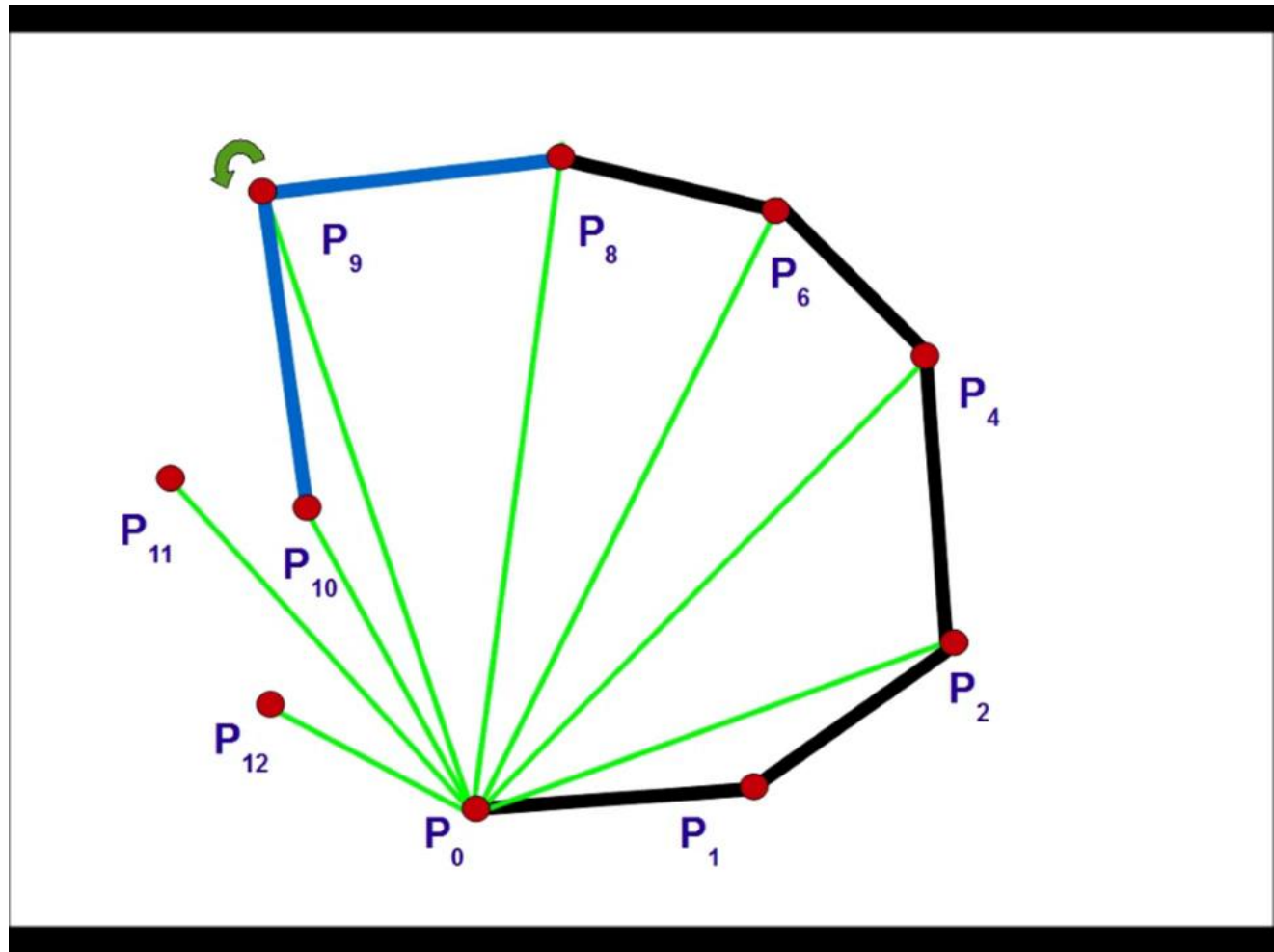
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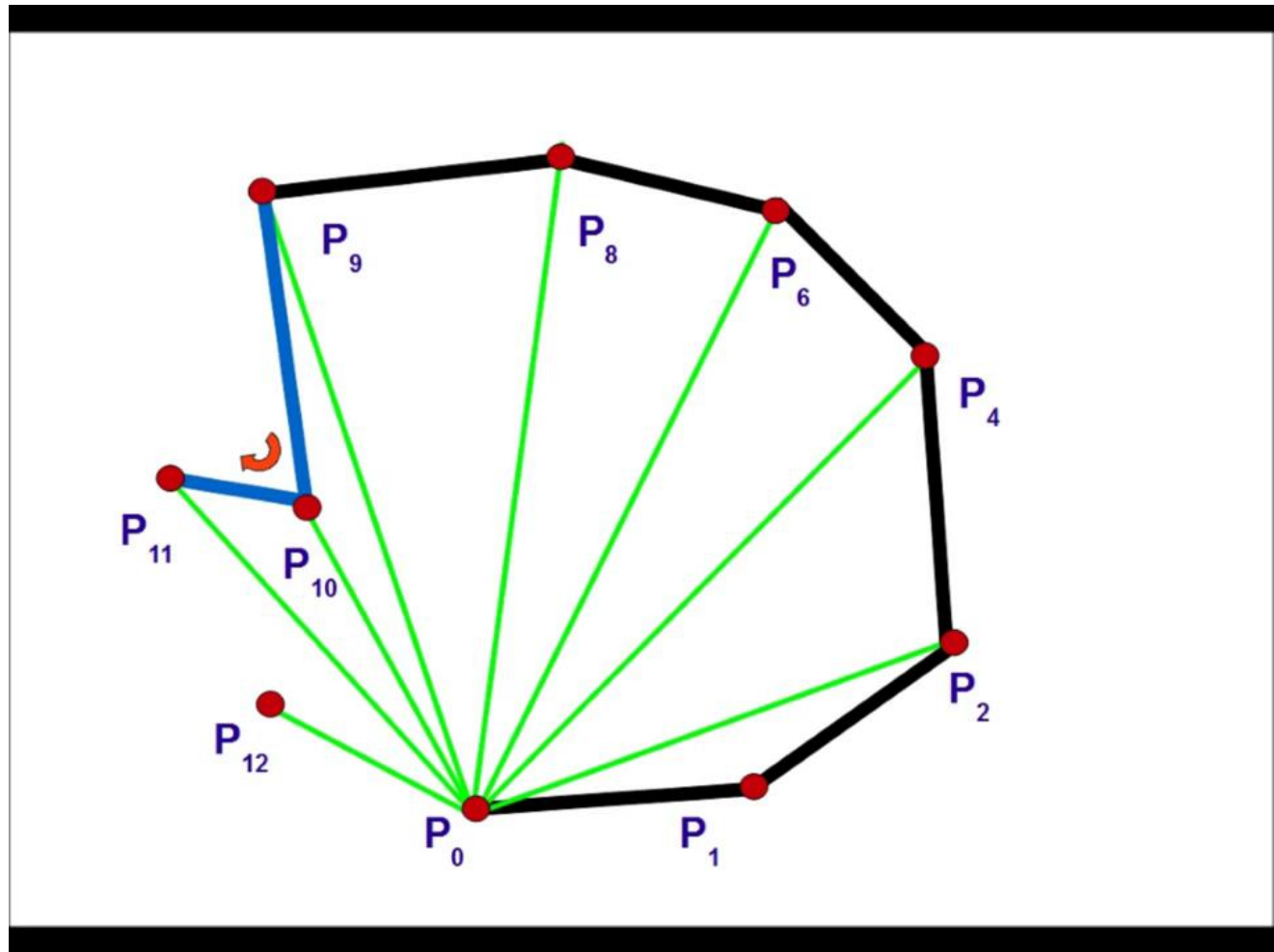
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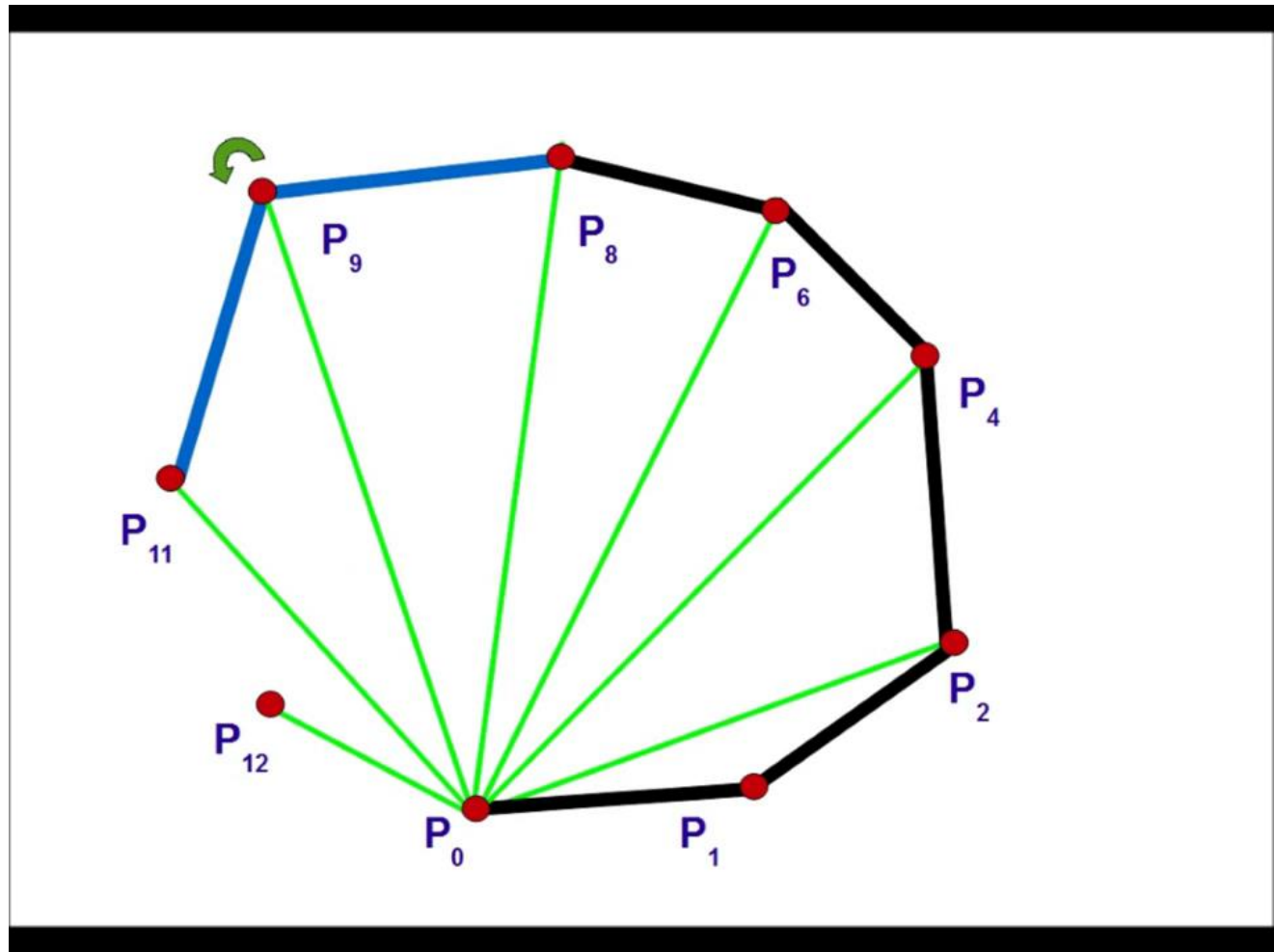
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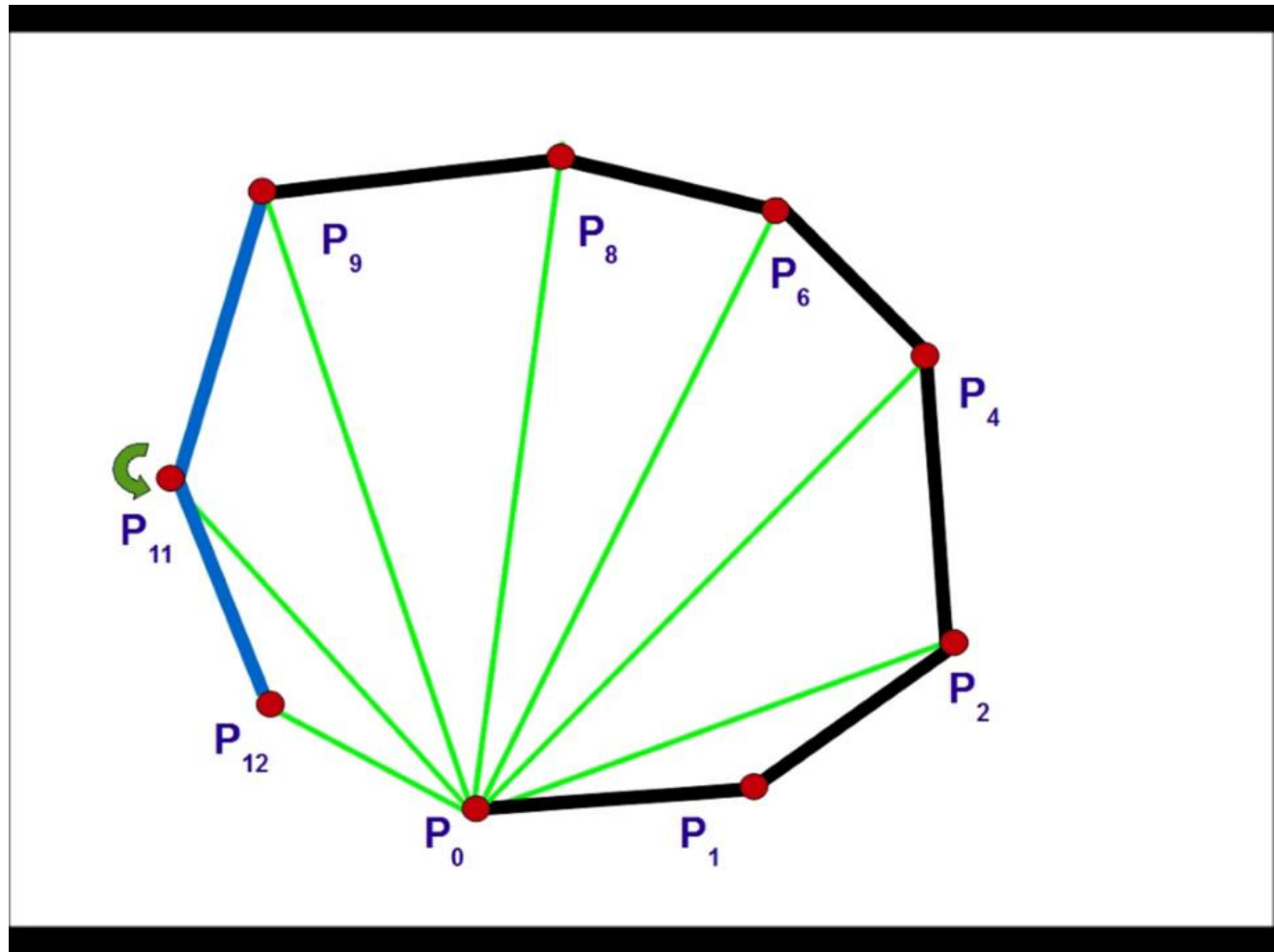
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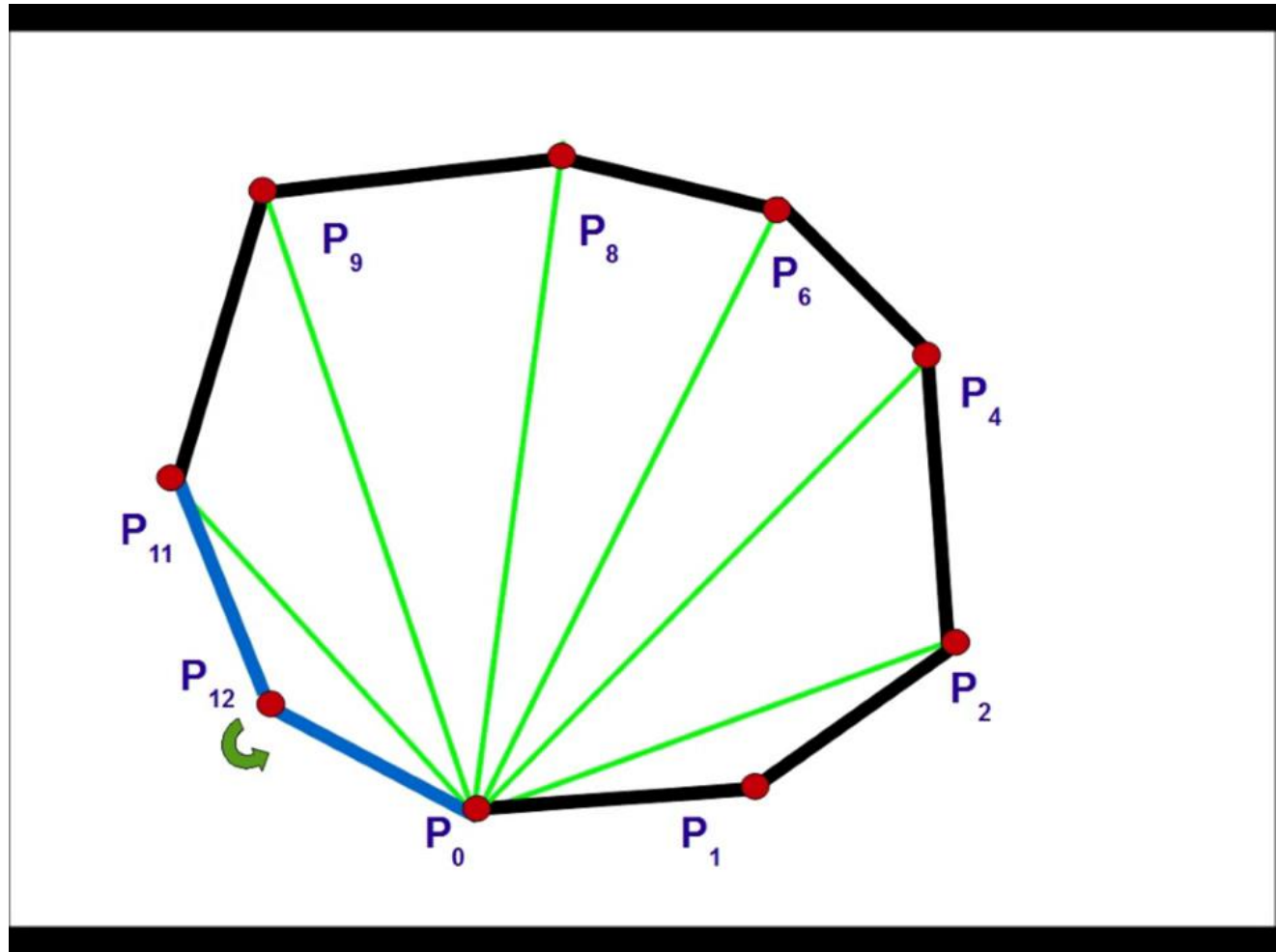
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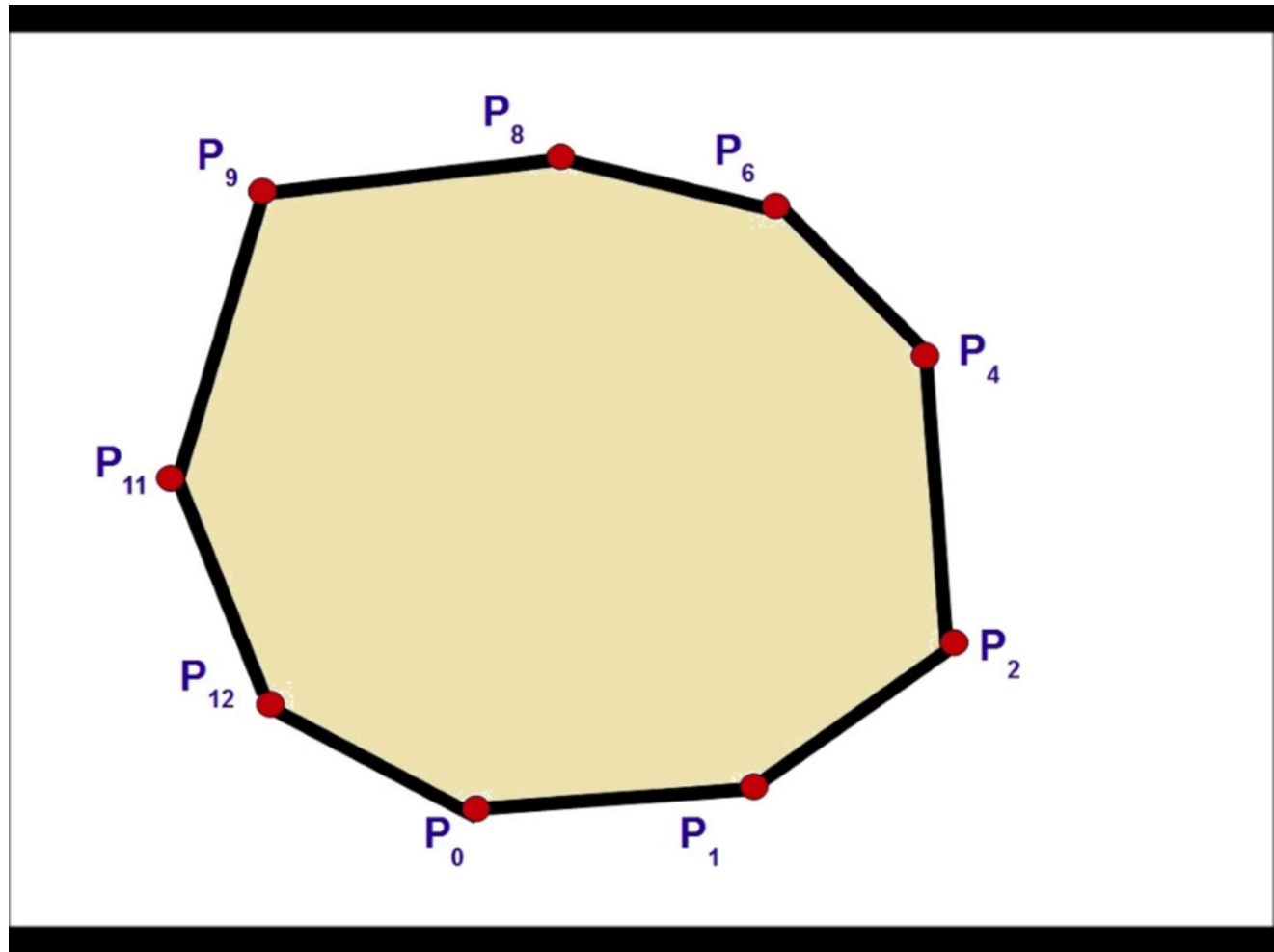


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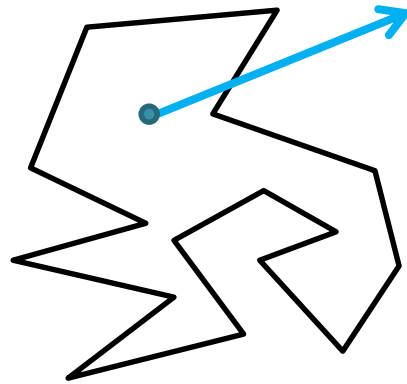


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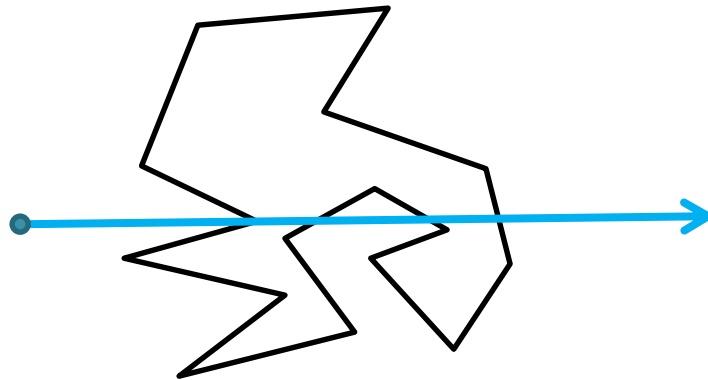
# Point in Polygon

- Extend point in random direction (forming a ray)
- Find number intersections with polygon.
- If even  $\rightarrow$  outside
- If odd  $\rightarrow$  inside



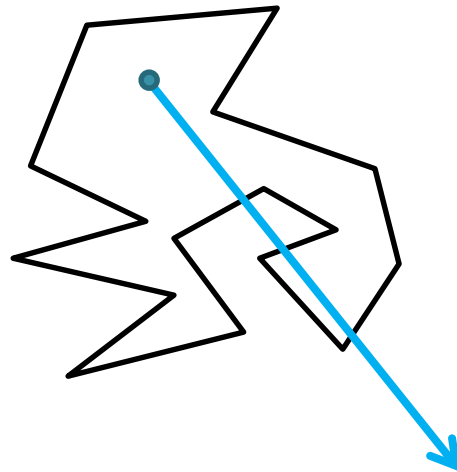
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# Area of Polygon

- Formula for the area of a polygon given  $n$  vertices in order:

$$A = \frac{1}{2} \sum_{i=0}^{n-1} (x_i \cdot y_{i+1} - x_{i+1} \cdot y_i)$$



**Questions?**